
Math 72 9.5 -1st

Math 62 11.5 -1st

Solve exponential
equations

Solve each equation.

a) Solve exactly.

b) Approximate solution to a) to 4 decimal places.

① $3^x = 7$

a) step 1: Take logs both sides using a log of base that you choose: base 10, base e, base 3.

BASE 10 Method

$$\log 3^x = \log 7$$

BASE e Method

$$\ln 3^x = \ln 7$$

Base 3 Method

$$\log_3 3^x = \log_3 7$$

step 2: Use log property to move exp x to front.

$$\log_b x^k = k \log_b x \text{ (move exp)}$$

$$\log_b b^x = x \text{ (inverse property)}$$

$$x \cdot \log 3 = \log 7$$

$$x \cdot \ln 3 = \ln 7$$

$$x \cdot \log_3 3 = \log_3 7$$

step 3: Isolate x.

$$\text{Notice } \log_3 3 = 1$$

$$x = \frac{\log 7}{\log 3}$$

$$x = \frac{\ln 7}{\ln 3}$$

$$x = \log_3 7$$

b). With base 10 and base e, plug directly into GC.

With base 3, use change of base formula

look at 5 places

1.77124

$$\approx \boxed{1.7712}$$

$$\log_3 7 = \frac{\log 7}{\log 3}$$

$$\text{or } \frac{\ln 7}{\ln 3}$$

M70

$$(2) e^{6x} = 5$$

a) BASE 10 Method

$$\log e^{6x} = \log 5$$

$$6x \log e = \log 5$$

$$x = \frac{\log 5}{6 \log e}$$

b) $x \approx .26823$

$$\boxed{x \approx .2682}$$

Base e Method * preferred
 $\ln e^x = x$
 inverse property

$$\ln e^{6x} = \ln 5$$

$$6x \cdot \ln e = \ln 5$$

$$6x = \ln 5$$

* Notice *
 $\ln e = 1$

$$\boxed{x = \frac{\ln 5}{6} = \frac{1}{6} \ln 5 = \ln \sqrt[6]{5}}$$

$$(3) 10^{2x+3} = 7$$

BASE 10 Method * preferred
 $\log 10^x = x$
 inverse property

a) $\log 10^{2x+3} = \log 7$

$$(2x+3) \cdot \log 10 = \log 7$$

* Notice $\log 10 = 1$ *

$$2x+3 = \log 7$$

$$2x = -3 + \log 7$$

$$\boxed{x = \frac{-3}{2} + \frac{\log 7}{2}}$$

$$\boxed{x = \frac{-3}{2} + \log \sqrt{7}}$$

b) $x \approx -1.07745$

$$\boxed{x = -1.0775}$$

Base e Method

$$\ln 10^{2x+3} = \ln 7$$

$$(2x+3) \ln 10 = \ln 7$$

$$2x+3 = \frac{\ln 7}{\ln 10}$$

$$2x = -3 + \frac{\ln 7}{\ln 10}$$

$$\boxed{x = \frac{-3}{2} + \frac{1}{2} \frac{\ln 7}{\ln 10}}$$

Aside: Are these expressions equal?

a) $\frac{\ln 5}{6}$

b) $\ln\left(\frac{5}{6}\right)$

c) $\frac{\ln 5}{\ln 6}$

Try in GC:

a) $\frac{\ln 5}{6} = \ln(5) / 6 \approx .2682$

b) $\ln\left(\frac{5}{6}\right) = \ln(5/6) \approx -.1823$

c) $\frac{\ln 5}{\ln 6} = \ln(5) / \ln(6) \approx .8982$

The short answer? No. They are totally different numerical results.

But what are these expressions if we use log properties or log facts?

b) $\ln\left(\frac{5}{6}\right) = \ln(5) - \ln(6)$

log property

c) $\frac{\ln 5}{\ln 6} = \log_6 5$

change of base

a) $\frac{\ln(5)}{6} = \frac{1}{6} \cdot \ln(5) = \ln(5^{\frac{1}{6}})$

log property

↑ this one is like $\frac{x}{6} = \frac{1}{6}x$

The moral: Be very careful where you close parentheses and where you write fraction bars.

$$(4) \quad 8 - 14 \cdot 5^{2x+1} = 1$$

a) step 0: Isolate the exponential 5^{2x+1}

$$-14 \cdot 5^{2x+1} = 1 - 8$$

$$\frac{-14 \cdot 5^{2x+1}}{-14} = \frac{-7}{-14}$$

$$5^{2x+1} = \frac{1}{2}$$

step 1: Take logs, step 2: Use log properties. step 3 isolate x

Base 10
Method

$$\log 5^{2x+1} = \log\left(\frac{1}{2}\right)$$

$$(2x+1) \cdot \log 5 = \log\left(\frac{1}{2}\right)$$

$$2x+1 = \frac{\log\left(\frac{1}{2}\right)}{\log(5)}$$

$$2x = -1 + \frac{\log\left(\frac{1}{2}\right)}{\log(5)}$$

$$x = -\frac{1}{2} + \frac{1}{2} \frac{\log\left(\frac{1}{2}\right)}{\log(5)}$$

Base e
Method

$$\ln 5^{2x+1} = \ln\left(\frac{1}{2}\right)$$

$$(2x+1) \ln 5 = \ln\left(\frac{1}{2}\right)$$

$$2x+1 = \frac{\ln\left(\frac{1}{2}\right)}{\ln(5)}$$

$$2x = -1 + \frac{\ln\left(\frac{1}{2}\right)}{\ln(5)}$$

$$x = -\frac{1}{2} + \frac{1}{2} \frac{\ln\left(\frac{1}{2}\right)}{\ln(5)}$$

Base 5
Method

$$\log_5 5^{2x+1} = \log_5\left(\frac{1}{2}\right)$$

$$2x+1 = \log_5\left(\frac{1}{2}\right)$$

$$2x = -1 + \log_5\left(\frac{1}{2}\right)$$

$$x = -\frac{1}{2} + \frac{1}{2} \log_5\left(\frac{1}{2}\right)$$

$$x = -\frac{1}{2} + \log_5 \sqrt{\frac{1}{2}}$$

b) -0.71533

$$\boxed{-0.7153}$$

$$\textcircled{5} \quad 4^{2x} = 3^{1-x}$$

Take logs both sides

$$\log 4^{2x} = \log 3^{1-x}$$

$$2x \cdot \log 4 = (1-x) \cdot \log 3$$

← I chose
base 10 logs, common log.
parentheses required

$$\underbrace{(2 \log 4)}_{\text{coefficient}} \cdot x = \underbrace{\log 3}_{\text{constant term}} - \underbrace{(\log 3)}_{\text{coefficient}} \cdot x$$

$$(2 \log 4)x + (\log 3)x = \log 3$$

Collect terms containing x
on the same side

$$x[2 \log 4 + \log 3] = \log 3$$

factor out x

$$a) \quad x = \frac{\log 3}{2 \log 4 + \log 3}$$

exact answer

$$= \frac{\log 3}{\log 4^2 + \log 3}$$

$$= \frac{\log 3}{\log 16 + \log 3}$$

$$= \frac{\log 3}{\log 16 \cdot 3}$$

$$x = \frac{\log 3}{\log 48} = x = \log_{48} 3$$

exact answer
combined using
log properties.

$$b). \quad x \approx .28379$$

$$x \approx .2838$$

Math 70 : Exponential Equations

- 1) How long does it take an investment for \$2000 to double if it is invested at 5% interest compounded quarterly? Round to the nearest tenth of a year.

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$P = 2000$$

$$A = 4000$$

$$r = .05$$

$$n = 4$$

$$t = ?$$

$$4000 = 2000 \left(1 + \frac{.05}{4}\right)^{4t}$$

solve for $t \Rightarrow$ exponent

step 0: Isolate exponential

$$\frac{4000}{2000} = \left(1 + \frac{.05}{4}\right)^{4t}$$

$$2 = (1.0125)^{4t}$$

step 1: Take log of both sides.

$$\log 2 = \log (1.0125)^{4t}$$

step 2: log property

$$\log 2 = 4t \cdot \log (1.0125)$$

step 3: Isolate t .

$$\frac{\log 2}{4 \log (1.0125)} = t$$

$$t = 13.94$$

$$\boxed{13.9 \text{ yr}}$$

- 2) Suppose that you invest \$1500 at an annual rate of 8% compounded monthly.

- a. Make a table giving the value of the investment at the end of each year for the next 14 years.

t	A
1	\$1624.50
2	\$1759.30
3	\$1905.40
4	\$2063.50
5	\$2234.80
6	\$2420.30
7	\$2621.10

t	A
8	\$2838.70
9	\$3074.30
10	\$3329.50
11	\$3605.80
12	\$3905.10
13	\$4229.20
14	\$4580.20

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 1500 \left(1 + \frac{.08}{12}\right)^{12t}$$

do not round! > frac

$$A = 1500 \left(\frac{151}{150}\right)^{12t}$$

- b. How long does it take the investment to triple? (Exact answer.)

$1500 \times 3 = 4500 \Rightarrow$ from table, between 13 and 14 years

$$A = 4500$$

$$P = 1500$$

$$r = .08$$

$$n = 12$$

$$t = \text{unknown}$$

$$4500 = 1500 \left(1 + \frac{.08}{12}\right)^{12t}$$

$$\frac{4500}{1500} = \left(\frac{151}{150}\right)^{12t}$$

$$3 = \left(\frac{151}{150}\right)^{12t}$$

$$\log 3 = \log \left(\frac{151}{150}\right)^{12t}$$

$$\log 3 = 12t \log \left(\frac{151}{150}\right)$$

$$\boxed{\frac{\log 3}{12 \log \left(\frac{151}{150}\right)} = t}$$

exact answer

$$\boxed{t \approx 13.77 \text{ yrs}}$$

- 3) The formula $P = 14.7e^{-0.21x}$ gives the average atmospheric pressure P , in pounds per square inch, at an altitude x , in miles above sea level. Use this formula to find the elevation of a Delta jet if the atmospheric pressure outside the jet is 7.5 pounds per square inch. Round to the nearest tenth.

$$7.5 = 14.7e^{-0.21x} \quad \text{find } x$$

$$\frac{7.5}{14.7} = e^{-0.21x}$$

$$\frac{25}{49} = e^{-0.21x}$$

$$\ln\left(\frac{25}{49}\right) = \ln(e^{-0.21x})$$

$$\ln\left(\frac{25}{49}\right) = -0.21x$$

$$x = \frac{\ln\left(\frac{25}{49}\right)}{-0.21} \approx 3.2044$$

3.2 miles above sea level

- 4) In 2010, the population of Illinois was approximately 12,830,000 and increasing by relative growth rate 0.5% annually. Assume that the population y continues to increase according to $y = y_0 e^{kt}$, where k is the annual relative growth rate and y_0 is the initial population. Predict how many years after which the population will be 13,500,000.

initial population $12,830,000 = y_0$

percent increase $= 0.5\% = 0.005 = k$

model $y = 12,830,000 e^{0.005t}$

"predict how many years" = find t .

population $13,500,000 = y$

$$13,500,000 = 12,830,000 e^{0.005t}$$

← This is an exponential equation.
• isolate the exponential
• take an appropriate log.

$$\frac{13,500,000}{12,830,000} = e^{0.005t}$$

$$\frac{1350}{1283} = e^{0.005t}$$

← base $e \Rightarrow$ use natural log, $\ln$

$$\ln\left(\frac{1350}{1283}\right) = \ln e^{0.005t} = 0.005t$$

← $\log_b b^x = x$ means $\ln e^x = x$

$$\boxed{\frac{\ln\left(\frac{1350}{1283}\right)}{0.005} = t} \quad \text{exact answer}$$

isolate t by $\div 0.005$

GC: $t \approx 10.1807$ during 2020 it passes 13.5 million
2021 entire year